Chapter 4:- Stellar magnitude

- ## Apparent magnitude

Apparent magnitude *m* of a star is a number that tells how bright that star appears at its great distance from Earth. The scale is logarithmic. Larger magnitudes correspond to fainter stars. Note that brightness is another way to say the flux of light, in Watts per square meter, coming towards us. The apparent magnitude (*m*) is given by:

$$m = c - 2.5\log_{10} B \text{ ....1}$$

Where *B* is the brightness and *c* is constant.

On this magnitude scale, a brightness ratio of 100 is set to correspond exactly to a magnitude difference of 5. As magnitude is a logarithmic scale, one can always transform a brightness ratio (B_2/B_1) into the equivalent magnitude difference (m_2-m_1) by the formula:

$$m_2 - m_1 = -2.5\log\frac{B_2}{B_1} \text{ ....2}$$

- ## Absolute Magnitude

Absolute magnitude (M_v) is the apparent magnitude the star would have if it were placed at a distance of 10 parsecs from the Earth. Doing this to a star (it is a little difficult), will either make it appear brighter or fainter. From the inverse square law for light, the ratio of its brightness at 10 pc to its brightness at its known distance *d* (in parsecs) is

$$\frac{B_{10}}{B_d} = \left(\frac{d}{10}\right)^2 \text{...............3}$$

Then, like the formula above, we say that its absolute magnitude is

$$M_v = m + 5 - 5\log d \text{ ....4}$$

Stars farther than 10 pc have (M_v) more negative than *m*, that is why there is a minus sign in the formula. If you use this formula, make sure you put the star's distance d in parsecs.

<u>Example: -</u> find the distance and parallax of star if its apparent magnitude is (25mag) and its absolute magnitude is (12mag).

• Bolometric Magnitude

The measured total of all radiation at all wavelengths from a star is called a bolometric magnitude. The corrections required to reduce visual magnitudes to bolometric magnitudes are large for very cool stars and for very hot ones but are relatively small for stars such as the Sun. A determination of the true total luminosity of a star affords a measure of its actual energy output. When the energy radiated by a star is observed at the Earth, only that portion to which the energy detector is sensitive and that can be transmitted through the atmosphere is recorded. Most of the energy of stars like the Sun is emitted in spectral regions that can be observed from the Earth's surface; but a cool dwarf star with a surface temperature of 3,000 K has an energy maximum on a wavelength scale at 10,000 Angstroms in the far-infrared, and most of its energy cannot therefore be measured as light. Bright, cool stars can be observed at infrared wavelengths, however, with special instruments that measure the amount of heat radiated by the star. Corrections for the heavy absorption of the infrared waves by water and other molecules in the Earth's air must be made unless an infrared payload has been lifted by balloon or rocket above the atmosphere.

• Star Colours and Temperatures

Stars appear to be exclusively white at first glance. But if we look carefully, we can notice a range of colours: blue, white, red, and even gold. The physics of blackbody radiation was enabled us to understand the variation of stellar colours. Shortly after blackbody radiation was understood, it was noticed that the spectra of stars look extremely similar to blackbody radiation curves of various temperatures, ranging from a few thousand Kelvin to ~50,000 Kelvin. The obvious conclusion is that stars are similar to blackbodies, and that the colour variation of stars is a direct consequence of their surface temperatures

• Stellar Luminosity

Luminosity is the amount of energy produced in a star and radiated into space per unit time in the form of E-M radiation.

The luminosity (L) of a star is proportional to the surface area times the energy radiated per square meter (σ is the Stefan-Boltzmann constant):

$$L = 4\pi R^2 \sigma {T_e}^4 \quad \ldots\ldots\ldots\ldots\ldots\ldots 5$$

Where R is the stellar radius and T_e is the temperature

Dividing the expressions for the luminosities of a star and the Sun allows us to compare them:

$$\frac{L_*}{L_\odot} = \left(\frac{R_*}{R_\odot}\right)^2 \left(\frac{T_{e_*}}{T_{e_\odot}}\right)^4 \quad \ldots\ldots\ldots\ldots\ldots.6$$

- **Luminosity-bolometric magnitude relation**

When a star at standard distance, (10 pc), there is a direct relation between its luminosity and brightness, so we can with equation (2) in term of luminosity (L) as

$$M_{bol_\odot} - M_{bol_*} = 2.5 \ \log\left(\frac{L_*}{L_\odot}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots7$$

- **Stellar energy-bolometric magnitude relation**

If the star is at a distance (r_*) from the observer, we can use the inverse square law to determine the amount of energy (E_*) that reaches the observer per unit area per unit time as below:

$$E_* = \frac{L_*}{4\pi r_*^4} \quad \ldots\ldots\ldots\ldots..8$$

$$E_* = \frac{R_*^2 \sigma T_{e*}^2}{r_*^2} \quad \ldots\ldots\ldots9$$

Since the star is too far, the angular diameter (α_*) is very small so:

$$\alpha_* = \frac{2R_*}{r_*} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots10$$

Substitute eq.(10) in eq.(9)

$$E_* = \frac{1}{4}\alpha_*^2 \sigma T_{e*}^4 \quad \ldots\ldots\ldots\ldots..11$$

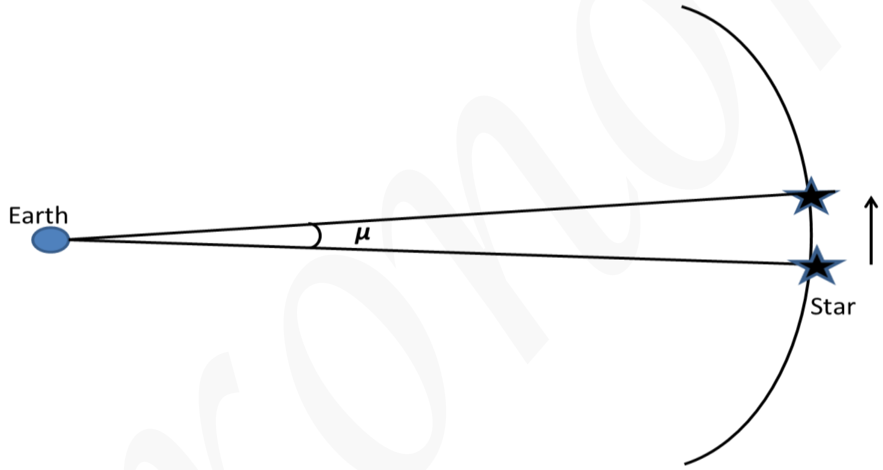
$$\frac{E_*}{E_\odot} = \left(\frac{\alpha_*}{\alpha_\odot}\right)^2 \left(\frac{T_{e*}}{T_{e\odot}}\right)^4 \quad \ldots\ldots\ldots\ldots\ldots..12$$

Using the equations (4, 7, 8, and 12), we find:

$$m_{bol\odot} - m_{bol*} = 5\log\left(\frac{\alpha_*}{\alpha_\odot}\right) + 10\log\left(\frac{T_{e*}}{T_{e\odot}}\right) \ldots\ldots\ldots 13$$

- **Stellar Motion**
  - **Proper Motion(μ):-**

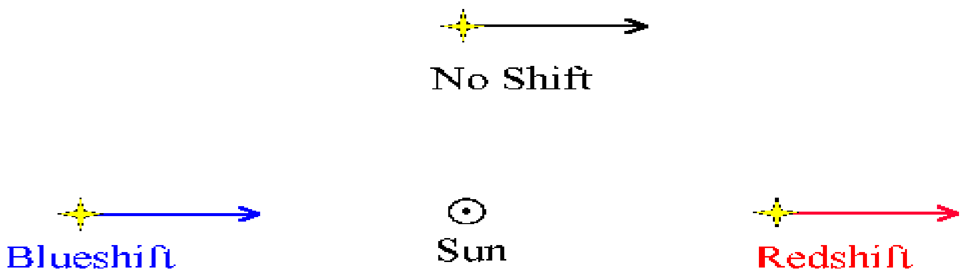
Proper motion is the angular change in position of a star across our line of sight, measured in arc seconds per year, and symbolized with the Greek letter μ.



Proper motion is not large. Typical proper motion is ~0.1 arcsec/year. The star with the largest proper motion is called Barnard's star. It moves 10.3 arcsec per year. Proper motion is generally measured by taking photographs several years apart and measuring the movement of the image of a star with respect to more distant background stars over that time period. Usually decades must elapse between successive photographs before a reliable measurement can be made

➢ Radial velocity

The **radial velocity** of a star is how fast it is moving directly towards or away from us.



Radial velocities are measured using the **Doppler Shift** of the star's spectrum:

- Star moving towards Earth: **Blueshift**
- Star moving away from Earth: **Redshift**
- Star moving across our line of sight: **No Shift**

In all cases, the **Radial Velocity is Independent of Distance**.

➢ Tangential Motion:-

It is the motion of star perpendicular to the line of sight. It is given in the relation below:

$$v_t = \frac{\mu r}{6.52 \times 10^{12}} \quad Km/sec$$

Where r is the distance between the Sun and the star in (Km), μ is the proper motion of star.

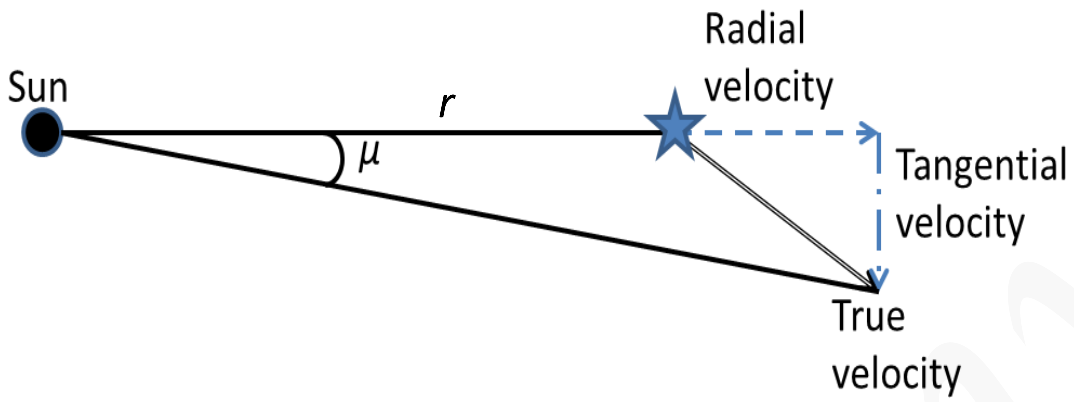
It can be determined by the relation below if (r) is given by parsec

$$v_t = 4.74\,\mu r = 4.74\frac{\mu}{\rho'} \quad Km/sec$$

➢ True Space Motions

The quantity we really want to know is the true motion of the star through space in 3-dimensions.

To find the true space velocity of a star, we need to break its motions into two velocity components:



Radial Velocity (v_r)

Measure this using the Doppler Shift of its spectrum.

Tangential Velocity (v_t)

Measure this from its Proper Motion and Distance:

$$v_t = 4.74\ \mu r$$

where:

μ = Proper Motion in arcsec/yr

r = Distance in parsecs

The formula above gives v_t in km/sec.

Each of these velocities forms the legs of a right triangle with the true space velocity (v) as the hypotenuse.

We can then use the Pythagorean Theorem to derive the True Space Velocity (v):

$$\begin{aligned} v^2 &= v_r^2 + v_t^2 \\ &= v_r^2 + (4.74\mu r)^2 \end{aligned}$$

To estimate the true space velocity, you need to measure three observable quantities:

- The Radial Velocity

- The Proper Motion
- The Distance

The last is often the most difficult to measure (as always).

• Stellar diameter:-

We can determine the diameter of a star using Stefan-Boltzmann law as below:

$$R_* = \sqrt{\frac{L_*}{\pi\sigma}} \times \frac{1}{2T_{e*}^2}$$

Usually we determine the diameter in term of solar diameter as below

$$\frac{R_*}{R_\odot} = \sqrt{\frac{L_*}{L_\odot}}\left(\frac{T_{e\odot}}{T_{e*}}\right)^2$$

• Stellar distance

The parallax is used to determine the distance for the nearby stars (up to 20 parsec). But for more distant star we use:

1- **Stellar motions**: All stars are in motion, but only for nearby stars are these motions perceivable. Statistically, therefore, the stars that have larger motions are nearer. By measuring the motions of a large number of stars, we can estimate their average distance from their average motion.

2- **Moving clusters**: Clusters of stars travel together, such as the Pleiades or Hyades star clusters. Analyzing the apparent motion of the cluster can give us the distance to it.

3- **Inverse-square law**: The apparent brightness of a star depends both on its intrinsic brightness (its luminosity, or how bright it really is) and its distance from us. If we know the luminosity of a star (for instance, we have a measured parallax for one star of the same type and know that others of the same type will have similar luminosities), we can measure its apparent brightness (also called its apparent magnitude) and work out the distance using the inverse-square law. There are several variations on this, many of which are used to measure distances to stars in other galaxies.

4- **Interstellar lines**: The space between stars is not empty, but contains a sparse distribution of gas. Some times this leaves absorption lines in the spectrum we observe from stars beyond the interstellar gas. The further a star is, the more absorption will be observed since the light has passed through more of the interstellar medium.

5- **Period-luminosity relation:** Some stars are regular pulsators. The physics of their pulsations is such that the period of one oscillation is related to the luminosity of the star. If we measure the period of such a star, we calculate its luminosity. From this, and its apparent magnitude, we can calculate the distance.

• Stellar spectra

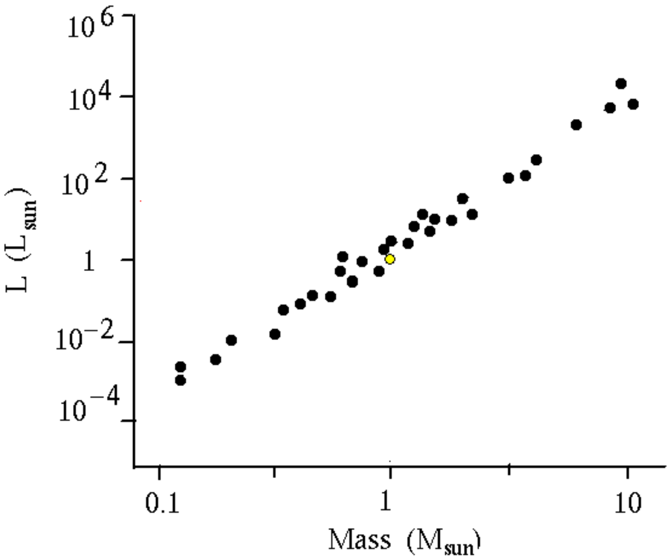
The absorption features present in stellar spectra allow us to divide stars into several spectral types depending on the temperature of the star. The scheme in use today is the Harvard spectral classification scheme. This sequence (**OBAFGKM**) is ordered from the hottest to the coolest stars as in the table below:

symbol	temperature	colour
O	≥ 33,000 K	blue
B	10,000–33,000 K	blue white
A	7,500–10,000 K	white
F	6,000–7,500 K	yellow white
G	5,200–6,000 K	yellow
K	3,700–5,200 K	orange
M	2,000–3,700 K	red
L	1,300–2,000 K	red brown
T	700–1,300 K	brown
Y	≤ 700 K	dark brown

Within each spectral type there are significant variations in the strengths of the absorption lines, and each type has been subdivided into 10 sub-classes numbered 0 to 9.

- **Mass-Luminosity Relation**

Observations of thousands of main sequence stars show that there is definite relationship between their mass and their luminosity. The more massive main sequence stars are hotter and more luminous than the low-mass main sequence stars as in the figure below:



The relation is given by

$$L_* \propto M^{3.5}$$

- **Hertzsprung - Russell diagram**

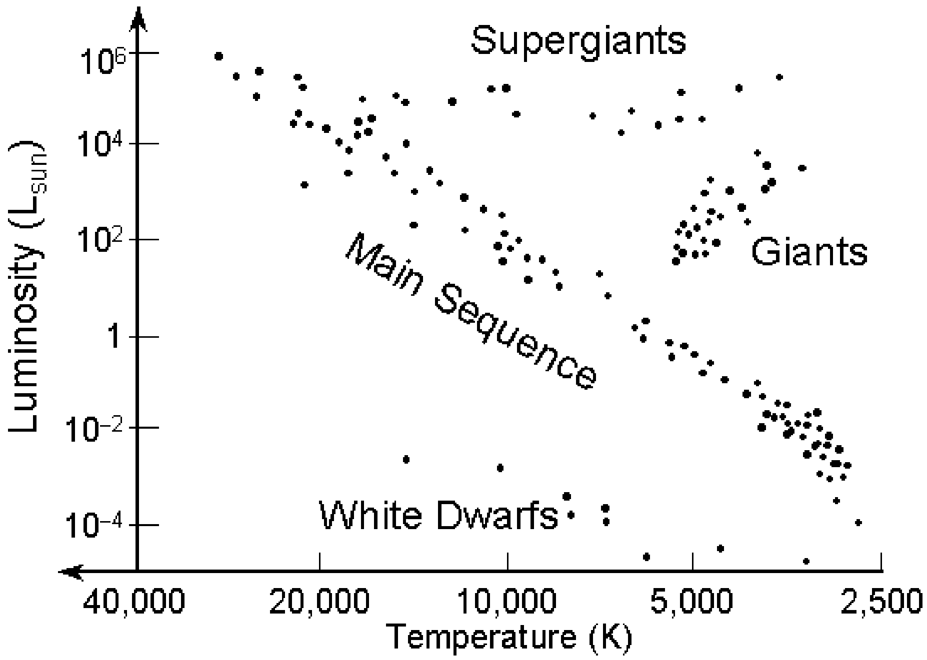
The Hertzsprung - Russell diagram is a tool that shows relationships and differences between stars. It is something of a "family portrait." It shows stars of different ages and in different stages, all at the same time. But it is a great tool to check your understanding of the star life cycle.

In the Hertzsprung-Russell (HR) Diagram, each star is represented by a dot. There are lots of stars out there, so there are lots of dots. The position of each dot on the diagram tells us two things about each star: its luminosity (or absolute magnitude) and its temperature.

The horizontal axis represents the star's surface temperature (not the star's core temperature – we cannot see into the core of a star, only its surface)! Usually this is labelled using the Kelvin temperature scale. But notice: In most graphs and diagrams, zero (or the smaller numbers) exists to the left on the diagram. This is not the case here.

On this diagram, the higher (hotter) temperatures are on the left, and the lower (cooler) temperatures are on the right. Some HR diagrams include the colour of stars as they can be seen through filters on spectrophotometers. This is also a "ratio scale."

A star in the upper left corner of the diagram would be hot and bright. A star in the upper right corner of the diagram would be cool and bright. The Sun rests approximately in the middle of the diagram, and it is the star which we use for comparison. A star in the lower left corner of the diagram would be hot and dim. A star in the lower right corner of the diagram would be cold and dim.



1- Main sequence stars

Most of the stars in the Universe are in the main sequence stage of their lives, a point in their stellar evolution where they're converting hydrogen into helium in their cores and releasing a tremendous amount of energy.

The main sequence is sometimes divided into upper and lower parts, based on the dominant process that a star uses to generate energy. Stars below about 1.5 solar masses primarily fuse hydrogen atoms together in a series of stages to form helium, a sequence called the proton–proton chain. Above this mass, in the upper main sequence, the nuclear fusion process mainly uses atoms of carbon, nitrogen and oxygen as intermediaries in the CNO cycle that produces helium from hydrogen atoms.

a- It does obey the mass-luminosity relation
b- Mass ($0.1M_\odot \rightarrow 50\, M_\odot$).

2- Giant Stars

A giant star is a star with substantially larger radius and luminosity than a main-sequence star of the same surface temperature. So they lie above the main sequence on the Hertzsprung–Russell diagram. Giant stars have radii up to a few hundred times the sun and luminosities between 10 and a few thousand times that of the Sun. Masses of giants and supergiants may be 10 to 30 times that of the Sun, but their volumes are often 10^6 to 10^7 times greater. Thus, they are low-density "diffuse" stars.

A star can become a type of giant star called a red giant when it has converted all the hydrogen fuel in its core into helium and begins burning up the hydrogen in its outer layers to resist gravity's inward pressure. As this happens, the star swells greatly in size and it glows more brightly. Due to subsequent changes in the fusion reactions, the star may move in and out of the red giant state, becoming different types of giant star. Giant stars do not obey the mass-luminosity relation. A red giant is a luminous giant star of low or intermediate mass (roughly 0.3–8 solar masses ($M_\odot$)) in a late phase of stellar evolution.

3- White dwarfs

When they reach the end of their long evolutions, smaller stars (those up to eight times as massive as our own sun) typically become white dwarfs. These ancient stars are incredibly dense. A teaspoonful of their matter would weigh as much on Earth as an elephant (5.5 tons). White dwarfs typically have a radius just .01 times that of our own sun, but their mass is about the same. White dwarfs are stars that have burned up all of the hydrogen they once used as nuclear fuel. These stars do not obey the mass-luminosity relation.

• H-R diagram for constant radii

We can represent H-R diagram in term of luminosity and the effective temperature with constant radius. Using Stefan-Boltzmann law:

$$L_* = 4\pi R_*^2 \sigma T_{e*}^4$$

Taking logs, we find

$$\log L_* = 4\log T_{e*} + 2\log R_* \log(4\pi\sigma)$$

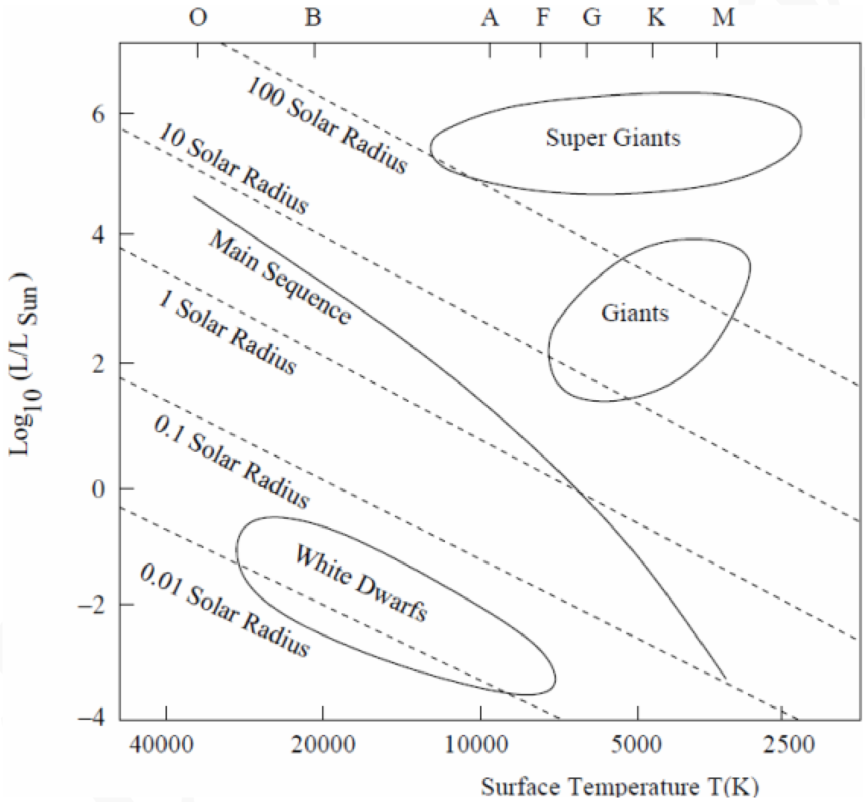
For the Sun

$$\log L_\odot = 4\log T_{e\odot} + 2\log R_\odot \log(4\pi\sigma)$$

By subtracting we get

$$\log\frac{L_*}{L_\odot} = 4\log\frac{T_{e*}}{T_\odot} + 2\log\frac{R_*}{R_\odot}$$

The above equation is a straight line equation ($y=mx+c$). Therefore objects of constant radius lie along straight lines in the H-R diagrams which slope from upper-left to lower-right, as in the figure below:



- **Life time of stars**

The life time of star (τ_*) is directly proportional with its mass (m) and inversely proportional with its (L_*), i.e.

$$\tau_* \propto \frac{m_*}{L_*} \Longrightarrow \tau_* \propto \frac{m_*}{m_*^{3.5}} \Longrightarrow \tau_* \propto \frac{1}{m_*^{2.5}}$$

The above relation is applied for main sequence stars. It expresses why there is a large number of faint stars and a little number of luminous stars.

- ## Stellar Evolution

The mass of the star determines what happens after the main sequence phase. Stars similar in mass to the Sun convert hydrogen into helium in their centres during the main-sequence phase, but eventually there is not enough hydrogen left in the centre for fusion to provide the necessary radiation pressure to balance gravity. The centre of the star contracts until it is hot enough for helium to be fused into carbon. The hydrogen in a shell continues to convert to helium, but the outer layers of the star have to expand to conserve energy. This makes the star appear brighter and cooler, and it becomes a red giant. During the red giant phase, a star often loses many outer layers, which are blown away by the radiation coming from below. Eventually, in the more massive stars of the group, the carbon may convert to even heavier elements, but eventually the energy generation will fizzle out and the star will collapse to a white dwarf. Astronomers think that white dwarfs ultimately cool to become black dwarfs.

Stars having masses between 0.08 and 0.4 times that of the Sun can have main sequence lifetimes greater than the current age of the Universe. These are known as red dwarfs, and are quite plentiful in the Universe.

There are very few stars with masses greater than five times the mass of the Sun, but their evolution ends in a spectacular fashion. They finish their main sequence lifetime in a way similar to the lower-mass stars, but become brighter and cooler on the outside and are called red supergiants. Carbon burning can develop at the star's centre and a complex set of element-burning shells can develop towards the end of the star's life. During this stage, many different chemical elements will be produced in the star and the central temperature will approach temperatures between 100 million and 600 million K. During this stage, the structure can resemble an onion skin with progressive layers (going inward) dominated by elements with greater and greater atomic mass. This process ends when the core is composed primarily of iron. For all the elements up to iron, the addition of more nucleons to the nucleus produces energy, and so yields a small contribution to the balance inside the star between gravity and radiation. To add more nucleons to the iron nucleus requires an input of energy, and so, once the centre of the star consists of iron, no more energy can be extracted. The star's core then has no resistance to the force of gravity, and once it starts to contract a very rapid collapse will take place. The protons and electrons combine to give a core composed of neutrons, and a vast amount of gravitational energy is released. This energy is sufficient to blow away all the outer parts of the star in a violent explosion, and the star becomes a supernova. The light of this one star at its peak during the explosion is then about as bright as the collective light from all the other 100 billion stars in the host galaxy. During this explosive phase, all the elements with atomic weights greater than iron are formed and, together with the rest of the outer regions of the star, are blown out into interstellar space. The central core of neutrons is left as a neutron star. We may observe some neutron stars as pulsars. This is remarkable, because in the early Universe there were no elements heavier than helium. The first stars were composed almost entirely of hydrogen and helium and there was no oxygen, nitrogen, iron, or any of the other elements that are necessary for life. These were all produced inside massive stars and were all spread throughout space by such supernovae events. We are made up of material that has been processed at least once inside stars.

- **Neutron Stars**

Neutron stars are compact objects that are created in the cores of massive stars during supernova explosions. The core of the star collapses, and crushes together every proton with a corresponding electron turning each electron-proton pair into a neutron. The neutrons, however, can often stop the collapse and remain as a neutron star.

Neutron stars are fascinating objects because they are the densest objects known. They are only about 10 miles in diameter, yet they are more massive than the Sun. One sugar cube of neutron star material weighs about 100 million tons, which is about as much as a mountain.

Neutron stars are one of the possible ends for a star. They result from massive stars which have mass greater than 4 to 8 solar mass. After these stars have finished burning their nuclear fuel, they undergo a supernova explosion. This explosion blows off the outer layers of a star into a beautiful supernova remnant. The central region of the star collapses under gravity. It collapses so much that protons and electrons combine to form neutrons.

- **Black Hole**

It is a great amount of matter packed into a very small area - think of a star ten times more massive than the Sun squeezed into a sphere approximately the diameter of New York City. The result is a gravitational field so strong that nothing, not even light, can escape. Black holes were predicted by Einstein's theory of general relativity, which showed that when a massive star dies, it leaves behind a small, dense remnant core. If the core's mass is more than about three times the mass of the Sun, the equations showed, the force of gravity overwhelms all other forces and produces a black hole.

Most black holes form from the remnants of a large star that dies in a supernova explosion. (Smaller stars become dense neutron stars, which are not massive enough to trap light.) If the total mass of the star is large enough (about three times the mass of the Sun), it can be proven theoretically that no force can keep the star from collapsing under the influence of gravity. However, as the star collapses, a strange thing occurs. As the surface of the star nears an imaginary surface called the "event horizon," time on the star slows relative to the time kept by observers far away. When the surface reaches the event horizon, time stands still, and the star can collapse no more - it is a frozen collapsing object.

Do they really exist?

It is impossible to see a black hole directly because no light can escape from them; they are black. But there are good reasons to think they exist.

When a large star has burnt all its fuel it explodes into a supernova. The stuff that is left collapses down to an extremely dense object known as a neutron star. We know that these objects exist because several have been found using radio telescopes.

If the neutron star is too large, the gravitational forces overwhelm the pressure gradients and collapse cannot be halted. The neutron star continues to shrink until it finally becomes a black hole. This mass limit is only a couple of solar masses, that is about twice the mass of our sun, and so we should expect at least a few neutron stars to have this mass.

A supernova occurs in our galaxy once every 300 years, and in neighbouring galaxies about 500 neutron stars have been identified. Therefore we are quite confident that there should also be some black holes.